

Bouncing Universe with Quintom Matter

Yi-Fu Cai^{a*}, Taotao Qiu^{a†}, Yun-Song Piao^b, Mingzhe Li^c, Xinmin Zhang^a

^a*Institute of High Energy Physics, Chinese Academy of Sciences, P.O. Box 918-4, Beijing 100049, P. R. China*

^b*College of Physical Sciences, Graduate School of Chinese Academy of Sciences, YuQuan Road 19A, Beijing 100049, China*

^c*Fakultät für Physik, Universität Bielefeld, D-33615 Bielefeld, Germany*

The bouncing universe provides a possible solution to the Big Bang singularity problem. In this paper we study the bouncing solution in the universe dominated by the Quintom matter with an equation of state (EoS) crossing the cosmological constant boundary. We will show explicitly the analytical and numerical bouncing solutions in three types of models for the Quintom matter with a phenomenological EoS, the two scalar fields and a scalar field with a modified Born-Infeld action.

I. INTRODUCTION

A bouncing universe with an initial contraction to a non-vanishing minimal radius, then subsequent an expanding phase provides a possible solution to the singularity problem of the standard Big Bang cosmology. For a successful bounce, it can be shown that within the framework of the standard 4-dimensional Friedmann-Robertson-Walker (FRW) cosmology with Einstein gravity the null energy condition (NEC) is violated for a period of time around the bouncing point. Moreover, for the universe entering into the hot Big Bang era after the bouncing, the EoS of the matter content w in the universe must transit from $w < -1$ to $w > -1$.

The Quintom model [1], proposed to understand the behavior of dark energy with an EoS of $w > -1$ in the past and $w < -1$ at present, has been supported by the observational data[2]. Quintom is a dynamical model of dark energy. It differs from the cosmological constant, Quintessence, Phantom, K-essence and so on in the determination of the cosmological evolution. A salient feature of the Quintom model is that its EoS can smoothly cross over $w = -1$. In the recent years there has been a lot of proposals for the Quintom-like models in the literature. In this paper we study the bouncing solution in the universe dominated by the Quintom matter and working with three specific models we will show explicitly the analytical and numerical solutions of the bounce.

We will start with a detailed examination on the necessary conditions required for a successful bounce. During the contracting phase, the scale factor $a(t)$ is decreasing, i.e., $\dot{a}(t) < 0$, and in the expanding phase we have $\dot{a}(t) > 0$. At the bouncing point, $\dot{a}(t) = 0$, and around this point $\ddot{a}(t) > 0$ for a period of time. Equivalently in the bouncing cosmology the hubble parameter H runs across zero from $H < 0$ to $H > 0$ and $H = 0$ at the bouncing point. A successful bounce requires around this point,

$$\dot{H} = -4\pi G\rho(1+w) > 0. \quad (1)$$

From (1) one can see that $w < -1$ in a neighborhood of the bouncing point.

After the bounce the universe needs to enter into the hot Big Bang era, otherwise the universe filled with the matter with an EoS $w < -1$ will reach the big rip singularity as what happens to the Phantom dark energy[3]. This requires the EoS of the matter to transit from $w < -1$ to $w > -1$.

In this paper, we study the bouncing solutions in the Quintom models. The paper is organized as follows. In section II, we present the analytical and numerical solutions for different types of models of the Quintom matter. Specifically we consider three models: i) a phenomenological Quintom fluid with a parameterized EoS crossing the cosmological constant boundary; ii) the two-field models of Quintom matter with one being the quintessence-like scalar and another the phantom-like scalar; iii) a single scalar with a Born-Infeld type action. III is the summary of the paper.

* caiyf@mail.ihep.ac.cn

† qiutt@mail.ihep.ac.cn

II. BOUNCING SOLUTION IN THE PRESENCE OF QUINTOM MATTER

A. A phenomenological Quintom model

We start with a study on the possibility of obtaining the bouncing solution in a phenomenological Quintom matter described by the following EoS:

$$w(t) = -r - \frac{s}{t^2} . \quad (2)$$

In (2) r and s are parameters and we require that $r < 1$ and $s > 0$. One can see from (2) that w runs from negative infinity at $t = 0$ to the cosmological constant boundary at $t = \sqrt{\frac{s}{1-r}}$ and then crosses this boundary.

Assuming that the universe is dominated by the matter with the EoS given by (2), we solve the Friedmann equation and obtain the corresponding evolution of hubble parameter $H(t)$ and scale factor $a(t)$ as follows,

$$H(t) = \frac{2}{3} \frac{t}{(1-r)t^2 + s} , \quad (3)$$

$$a(t) = \left(t^2 + \frac{s}{1-r}\right)^{\frac{1}{3(1-r)}} . \quad (4)$$

Here we choose $t = 0$ as the bouncing point and normalize $a = 1$ at this point. One can see that our solution provides a picture of the universe evolution with contracting for $t < 0$, and then bouncing at $t = 0$ to the expanding phase for $t > 0$. In Fig. 1 we plot the evolution of the EoS, the hubble parameter and the scale factor.

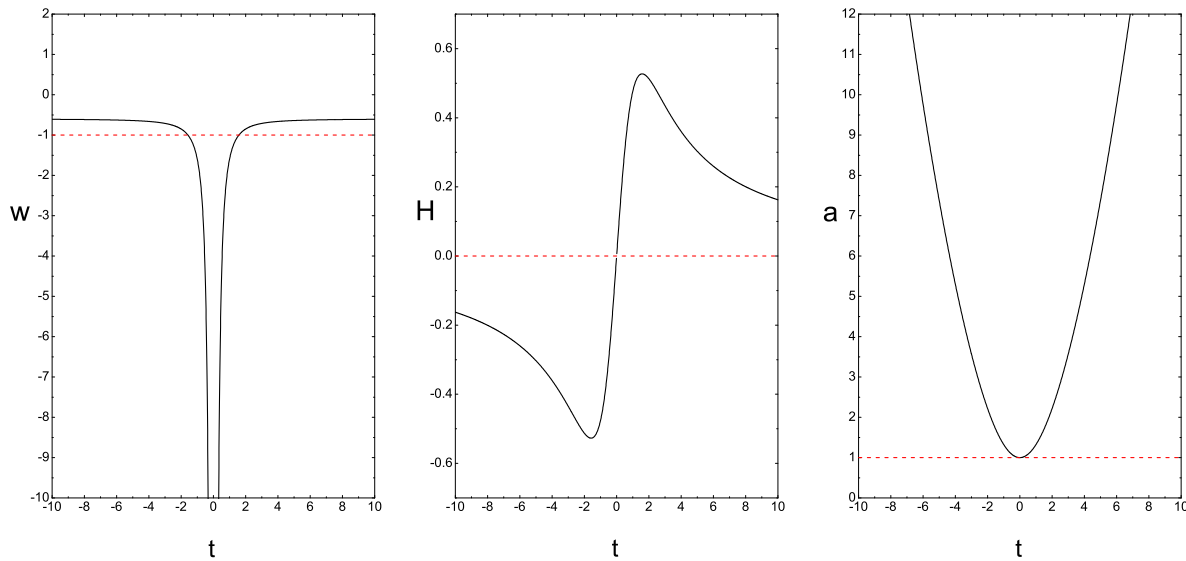


FIG. 1: Plot of the evolution of the EoS w , the hubble parameter H and the scale factor a as a function of the cosmic time t . Here in the numerical calculation we have taken $r = 0.6$ and $s = 1$.

One can see from Fig. 1 that a non-singular bouncing happens at $t = 0$ with the hubble parameter H running across zero and a minimal non-vanishing scale factor a . At the bouncing point w approaches negative infinity.

B. Two-field Quintom model

Having presented the bouncing solution with the phenomenological Quintom matter, we now study the bounce in the scalar field models of Quintom matter. However it is not easy to build a Quintom model theoretically. The No-Go theorem proven in Ref. [4] (also see Ref. [1, 5, 6, 7, 8, 9]) forbids the traditional scalar field model with a lagrangian of general form $\mathcal{L} = \mathcal{L}(\phi, \nabla_\mu \phi \nabla^\mu \phi)$ to have its EoS cross over the cosmological constant boundary. Therefore, to realize a viable Quintom field model in the framework of Einstein's gravity theory, it needs to introduce extra degree of freedom to the conventional theory with a single scalar field. The simplest Quintom model involves two scalars with one being the Quintessence-like and another the Phantom-like [1, 10]. This model has been studied in detail later on in the literature. In the recent years there have been a lot of activities in the theoretical study on Quintom-like models such as a single scalar with high-derivative [11, 12], vector field [13], extended theory of gravity [14] and so on, see e.g. [15].

In this section we consider a two-field Quintom model with the action given by

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} \partial_\mu \phi_1 \partial^\mu \phi_1 - \frac{1}{2} \partial_\mu \phi_2 \partial^\mu \phi_2 - V(\phi_1, \phi_2) \right], \quad (5)$$

where the metric is in form of $(+, -, -, -)$. Here the field ϕ_1 has a canonical kinetic term, but ϕ_2 is a ghost field. In the framework of FRW cosmology, we can easily obtain the energy density and the pressure of this model,

$$\rho = \frac{1}{2} \dot{\phi}_1^2 - \frac{1}{2} \dot{\phi}_2^2 + V, \quad p = \frac{1}{2} \dot{\phi}_1^2 - \frac{1}{2} \dot{\phi}_2^2 - V, \quad (6)$$

and the Einstein equations are given by

$$H^2 = \frac{8\pi G}{3} \left(\frac{1}{2} \dot{\phi}_1^2 - \frac{1}{2} \dot{\phi}_2^2 + V \right), \quad (7)$$

$$\ddot{\phi}_1 + 3H\dot{\phi}_1 + \frac{dV}{d\phi_1} = 0, \quad (8)$$

$$\ddot{\phi}_2 + 3H\dot{\phi}_2 - \frac{dV}{d\phi_2} = 0. \quad (9)$$

From Eq. (1), we can see that a bouncing solution requires $\dot{\phi}_2^2 = \dot{\phi}_1^2 + 2V$ when H crosses zero; and the Quintom behavior requires $\dot{\phi}_2^2 = \dot{\phi}_1^2$ when w crosses -1 . These constraints can be easily satisfied in the parameter space of this model.

In Fig. 2 and Fig. 3, we show the bouncing solution for two different type of potentials. In Fig. 2 we take $V(\phi_1, \phi_2) = V_1 e^{-\lambda_1 \frac{\phi_1^2}{M^2}} + V_2 e^{-\lambda_2 \frac{\phi_2^2}{M^2}}$. In the numerical calculation we normalize the dimensional parameters such as V_1, V_2, ϕ_1 and ϕ_2 by a mass scale M which we take specifically to be $10^{-2} M_{pl}$. And the hubble parameter is normalized with $\frac{M^2}{M_{pl}^2}$. One can see from this figure the non-singular behavior of the Hubble parameter and the scale factor for a bounce. The EoS w crosses over the $w = -1$ and approaches to negative infinity at the bouncing point. And due to the oscillatory behavior of the field ϕ_1 in the evolution, the EoS w is also oscillating around bouncing point.

In Fig. 3, we take $V(\phi) = \frac{1}{2} m^2 \phi_1^2 + V_0 \phi_2^{-2}$. This model also provides a bouncing solution, however the detailed evolution of the universe differs from the one shown in Fig. 2. Fig. 3 shows that the EoS of the Quintom matter will approach $w = 1$ asymptotically.

C. A single scalar with high-derivative terms

In this section we consider a class of Quintom models described by an effective lagrangian with higher derivative operators. Starting with a canonical scalar field with the lagrangian $\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi)$. This type of models has been considered as a candidate for dark energy, however as shown by the No-Go theorem it does not give w crossing -1 . As an effective theory we know that the lagrangian should include more operators, especially if these operators involve the term $\square\phi$, as pointed in Ref. [11] it will give rise to an EoS across $w = -1$. A connection of this type of Quintom theory to the string theory has been considered in Ref. [16] and [12]. In this paper we take the string-inspired model in [16] for the detailed study on the bouncing solution, where the action is given by

$$S = \int d^4x \sqrt{-g} \left[-V(\phi) \sqrt{1 - \alpha' \nabla_\mu \phi \nabla^\mu \phi} + \beta' \phi \square \phi \right]. \quad (10)$$

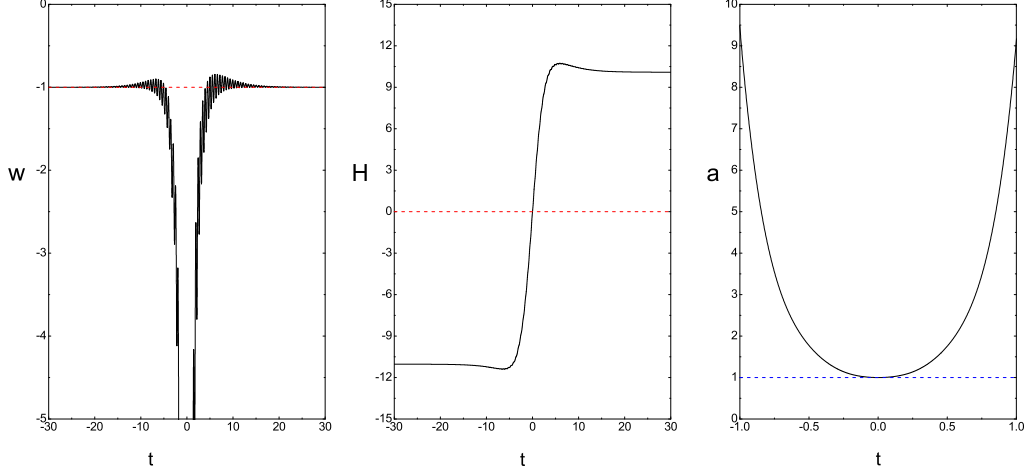


FIG. 2: The plots of the evolutions of the EoS w , hubble parameter H and the scale factor a . In the numerical calculation we choose $V(\phi_1, \phi_2) = V_1 e^{-\lambda_1 \frac{\phi_1^2}{M^2}} + V_2 e^{-\lambda_2 \frac{\phi_2^2}{M^2}}$ with parameters: $V_1 = 15$, $V_2 = 1$, $\lambda_1 = -1.0$, $\lambda_2 = 1.0$, and for the initial conditions $\phi_1 = 0.5$, $\dot{\phi}_1 = 0.1$, $\phi_2 = 0.3$, $\dot{\phi}_2 = 4$.

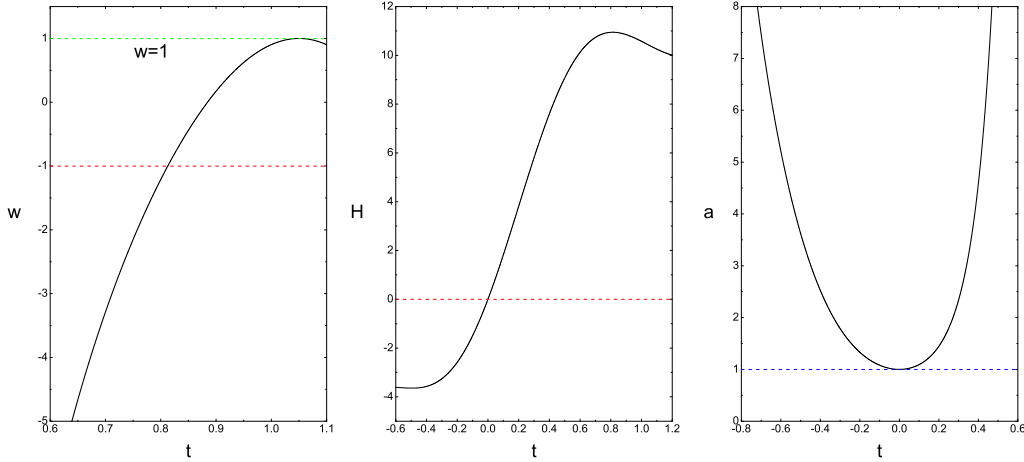


FIG. 3: The same plots as Fig. 2 with different potential and model parameters $V(\phi) = \frac{1}{2}m^2\phi_1^2 + V_0\phi_2^{-2}$, $m = 2$, $V_0 = 0.4$, and for the initial conditions $\phi_1 = 2$, $\dot{\phi}_1 = 3$, $\phi_2 = 1$, $\dot{\phi}_2 = 2$.

This is a generalized version of “Born-Infeld” action[17, 18] with the introduction of the β' term. To the lowest order, the Box-operator term $\phi\Box\phi$ is equivalent to the term $\nabla_\mu\phi\nabla^\mu\phi$ when the tachyon is on the top of its potential. However when the tachyon rolls down from the top of the potential, these two terms exhibit different dynamical behavior. The two parameters α' and β' in (10) could be arbitrary in the case of the background flux being turned on [19]. One interesting feature of this model is that it provides the possibility of its EoS w running across the cosmological constant boundary. In the analytical and numerical studies below to make two parameters (α' , β') dimensionless, it is convenient to redefine $\alpha = \alpha'M^4$ and $\beta = \beta'M^4$ where M is an energy scale of the effective theory of tachyon.

From (10) we obtain the equation of motion for the scalar field ϕ :

$$\frac{\beta}{2}\square\left(\frac{V\phi}{f}\right) + \alpha\nabla_\mu\left(\frac{V\nabla^\mu\phi}{f}\right) + M^4V_\phi f + \frac{\beta V}{2f}\square\phi = 0, \quad (11)$$

where $f = \sqrt{1 - \alpha'\nabla_\mu\phi\nabla^\mu\phi + \beta'\phi\square\phi}$ and $V_\phi = dV/d\phi$. Correspondingly, the stress energy tensor of the model is given by

$$T_{\mu\nu} = g_{\mu\nu}\left[Vf - \frac{\beta}{2M^4}\nabla_\rho\left(\frac{\phi V}{f}\nabla^\rho\phi\right)\right] + \frac{\alpha}{M^4}\frac{V}{f}\nabla_\mu\phi\nabla_\nu\phi + \frac{\beta}{2M^4}\nabla_\mu\left(\frac{\phi V}{f}\right)\nabla_\nu\phi + \frac{\beta}{2M^4}\nabla_\nu\left(\frac{\phi V}{f}\right)\nabla_\mu\phi. \quad (12)$$

Technically, it is very useful to define a parameter $\psi \equiv \frac{\partial\mathcal{L}}{\partial\square\phi} = -\frac{\beta\phi V}{2M^4f}$ to solve (11) and (12). In the framework of a flat FRW universe filled with a homogenous scalar field ϕ , we have the equations of motion in form of

$$\ddot{\phi} + 3H\dot{\phi} = \frac{\beta\phi}{4M^4\psi^2}V^2 - \frac{M^4}{\beta\phi} + \frac{\alpha}{\beta\phi}\dot{\phi}^2, \quad (13)$$

$$\ddot{\psi} + 3H\dot{\psi} = (2\alpha + \beta)\left(\frac{M^4\psi}{\beta^2\phi^2} - \frac{V^2}{4M^4\psi}\right) - \frac{\beta\phi}{2M^4\psi}VV_\phi - (2\alpha - \beta)\frac{\alpha\psi}{\beta^2\phi^2}\dot{\phi}^2 - \frac{2\alpha}{\beta\phi}\dot{\psi}\dot{\phi}. \quad (14)$$

Moreover, the energy density and the pressure of this field can be written as

$$\rho = -\frac{\alpha\psi}{\beta\phi}\dot{\phi}^2 - \dot{\psi}\dot{\phi} - \frac{\beta\phi}{4M^4\psi}V^2 - \frac{M^4\psi}{\beta\phi}, \quad (15)$$

$$p = -\frac{\alpha\psi}{\beta\phi}\dot{\phi}^2 - \dot{\psi}\dot{\phi} + \frac{\beta\phi}{4M^4\psi}V^2 + \frac{M^4\psi}{\beta\phi}. \quad (16)$$

From Eq.(1), one can see that a successful bounce requires:

$$\frac{\beta\phi}{4M^4\psi}V^2 + \frac{M^4\psi}{\beta\phi} = -\frac{\alpha\psi}{\beta\phi}\dot{\phi}^2 - \dot{\psi}\dot{\phi} < 0. \quad (17)$$

We will show below that (17) can be satisfied easily for our model. In the numerical study on the bouncing solution, we constrain the parameters α and β so that the model in (10) when expanding the derivative terms in the square root to the lowest order gives rise to a canonical kinetic term for the scalar field ϕ [16], i.e., $\alpha + \beta > 0$.

In Fig. 4 and Fig. 5 we show the bounce solution for different potentials. In Fig. 4, we take $V(\phi) = V_0e^{-\lambda\phi^2/M^2}$ with λ being a dimensionless parameter. One can see from this figure the scale factor initially decreases, then passes through its minimum and increases. Moreover, away from the bouncing point in the expanding phase, the EoS of the scalar field crosses $w = -1$ and approaches $w = -0.6$, which gives rise to a possible inflationary phase after the bouncing. In the numerical calculation we take the energy scale M to be $10^{-2}M_{pl}$, and the hubble parameter is normalized with $\frac{M^2}{M_{pl}}$.

In Fig. 5, we consider the model with potential $V(\phi) = V_0/\phi$ and then show another example of the bouncing solution¹. Here the energy scale M is chosen to be $10^{-2}M_{pl}$ as well. One can see from this figure the clear picture of the bouncing, however the detailed evolution of the universe differs from the one shown in Fig. 4. After entering the expanding phase, the EoS w crosses the cosmological constant boundary and approaches $w = \frac{1}{3}$, which is equivalent to the EoS of the radiation.

III. CONCLUSION AND DISCUSSIONS

In this paper we have studied the possibility of obtaining a non-singular bounce in the presence of the Quintom matter. In the literature there have been a lot of efforts in constructing the bouncing universe, for instance, the Pre Big Bang scenario[21], and the Ekpyrotic scenario[22]. In Refs. [23, 24, 25] and [26, 27] the authors have considered models with the modifications of gravity with the high order terms. In general these models modify the 4-dimensional

¹ A Born-Infeld lagrangian with this potential provides a scaling solution, see Ref. [20].

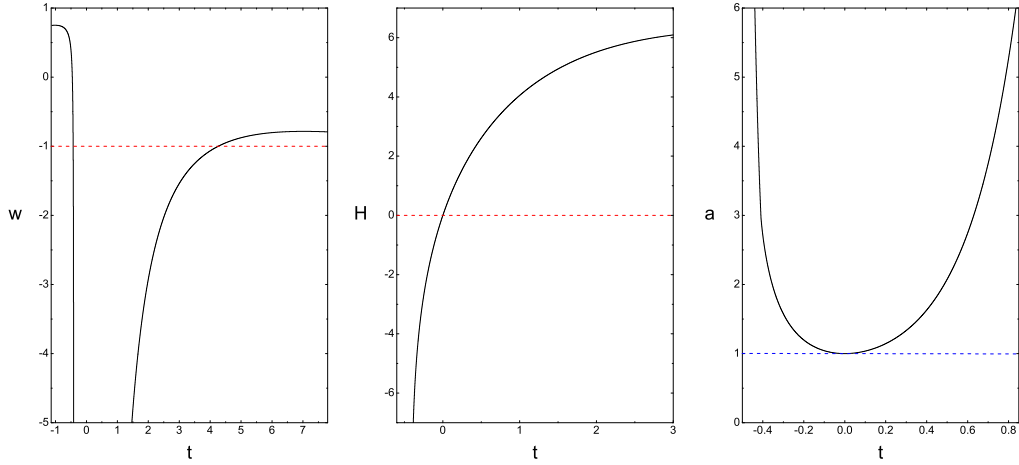


FIG. 4: The plots of the evolution of the EoS w , the hubble parameter H and the scale factor a . Here in the numerical calculation we take the potential $V(\phi) = V_0 e^{-\lambda \phi^2}$, $\alpha = -0.2$, $\beta = 2$, $\lambda = 2$, $V_0 = 5$, and the initial values are $\phi = 1$, $\dot{\phi} = 3$, $H = -1$, and $\psi = -80$.

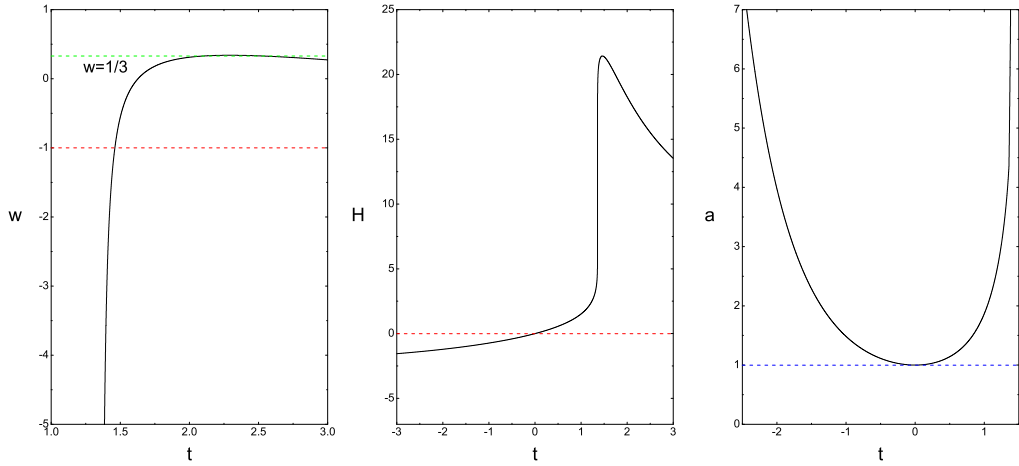


FIG. 5: The plots of the evolutions of the EoS w , hubble parameter H and the scale factor a . In the numerical calculation we choose the potential as $V(\phi) = \frac{V_0}{\phi}$, $\alpha = -0.2$, $\beta = 2$, $V_0 = 0.7$, and for the initial conditions $\phi = 10$, $\dot{\phi} = -3$, $H = -1$, $\psi = -40$.

Einstein gravity. However, the models we consider for bounce universe in this paper are restricted to be within the standard 4-dimensional FRW framework.

Recently two papers [28, 29] have studied the possibilities of having a bounce universe with the ghost condensate. In the original formulation the ghost condensate[30] will not be able to give EoS crossing $w = -1$. The authors of these papers[28, 29] have considered a generalized model of ghost condensate[31] and shown the bouncing solutions. In this paper we have studied the general issue of obtaining a bouncing universe with the Quintom matter. Our results show that a universe in the presence of the Quintom matter will avoid the problem of the Big Bang singularity. Explicitly for the analytical and numerical studies we have considered three models: the phenomenological model,

the two-field model and the string-inspired Quintom model. The latter one is a generalization of the idea in Ref.[11] by introducing higher derivative terms to realize the EoS crossing $w = -1$. In this regard, this model for the bounce solution has the similarity with a recent paper [32] where the authors presented a bouncing solution with non-local SFT[12].

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